

R15

Code No: 123AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, September/October - 2023

PROBABILITY AND STATISTICS

(Common to ME, CSE, IT, MCT, AME, MIE)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

- 1.a) Suppose X has the following probability mass function $p(0) = 0.2, p(1) = 0.5, p(2) = 0.3$. Calculate $E(X^2)$. [2]
- b) Calculate $\text{Var}(X)$ when X represents the outcome when we roll a fair die. [3]
- c) If X and Y have joint p.d.f $(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{Otherwise} \end{cases}$, Check whether X and Y are independent or not. [2]
- d) Let X and Y have joint density function $f(x, y) = 2, 0 < x < y < 1$. Find the conditional function of Y given $X = x$. [3]
- e) Write any two characteristics of χ^2 test? [2]
- f) Explain the terms sample size and sampling error in a random sampling? [3]
- g) Explain the notation to represent a Queuing system. [2]
- h) Define traffic intensity of a Queue. [3]
- i) Define the State space. [2]
- j) Give an example of stationary processes and justify your claim? [3]

PART - B

(50 Marks)

- 2.a) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 50 and variance 25.
 - (i) What can be said about the probability that this week's production will exceed 75?
 - (ii) What can be said about the probability that this week's production will be between 40 and 60?
- b) Let X be a random variable with $E(X) = 10$ and $\text{Var}(X) = 25$. Find the positive values of 'a' and 'b' such that Y has expectation 0 and variance 1. [5+5]

OR

- 3.a) The number of PCs sold daily at a computer world is uniformly distributed with a minimum of 2000 PC and a maximum of 5000 PCs. Find
 - (i) The probability that daily sales will fall between 2500 and 3000
 - (ii) What is the probability that the computer world will sell at least 4000 PCs?
 - (iii) What is the probability that the computer world will exactly sell 2500 PCs?
- b) If a random variable 'x' has the M.G.F $M_x(t) = \frac{2}{2-t}$, find the variance of 'x'. [7+3]

- 4.a) 3 balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denote the number of red balls. Find the P(X,Y).

- b) The joint p.d.f of random variable X and Y is given by $f(x, y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of K and prove also that X and Y are independent. [5+5]

OR

- 5.a) Calculate the correlation coefficient for the following heights(in inches) of fathers X and their sons Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

- b) Let X, Y and Z be uncorrelated random variable with 0 means and standard deviations 5,12 and 9 respectively. If $U = X+Y$ and $V = Y+Z$, Find Karl-Pearson's coefficient of correlation between U and V. [5+5]

- 6.a) The mean life of sample of 25 bulbs is found to be 1550 hrs with S.D of 120 hrs. The company manufacturing bulbs claims that the average life of the bulbs is 1600 hrs. Is the claim acceptable at 95% level of significance.

- b) In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this State at 1% level of significance? [5+5]

OR

- 7.a) Two independent samples of sizes 9 and 7 from a normal population at the following values of the variables.

Sample 1	18	13	12	15	12	14	16	14	15
Sample 2	16	19	13	16	18	13	15		

Do the estimates of the population variances differ significantly at 5% level?

- b) In 120 throws of a single die the following distribution of faces were observed.

Faces	1	2	3	4	5	6
Frequency	30	25	18	10	22	15

Can you say that the die is biased?

8. Explain (M/M/1) : (∞/FCFS) queueing model. [10]

OR

9. A Supermarket has a server servicing at the counter. The customer arrive in a Poisson fashion at the rate of 10 per hour. The service time for each customer is expected with mean 4 minutes. Find:

- a) The probability of a customer has to wait for the service,
 b) Average queue length and
 c) The average time spent by a customer in the queue. [10]

- 10.a) Explain briefly the main characteristics of Queuing system

- b) Derive the formula for the probability of having n customers in the system. [5+5]

OR

11. The Probability distribution of the process $\{X(t)\}$ is given by

$$P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, \dots \\ \frac{at}{(1+at)}, & n = 0 \end{cases}, \text{ Show that it is not stationary. [10]}$$